Real-Time Thermospheric Density Estimation Via Two-Line-Element Data Assimilation

David J. Gondelach¹, and Richard Linares¹

¹Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, USA

Key Points:

• Thermospheric density is estimated using a dynamic reduced-order thermosphere model and two-line element data.
• The two-line element data is assimilated using a Kalman filter to provide both density and uncertainty estimates.
• The estimated densities are validated against CHAMP and GRACE accelerometer-derived density data.

Corresponding author: David Gondelach, http://orcid.org/0000-0002-8511-9523, dgondela@mit.edu
Abstract

Inaccurate estimates of the thermospheric density are a major source of error in low Earth orbit prediction. Therefore, real-time density estimation is required to improve orbit prediction. In this work, we develop a dynamic reduced-order model for the thermospheric density that enables real-time density estimation using two-line element (TLE) data. For this, the global thermospheric density is represented by the main spatial modes of the atmosphere and a time-varying low-dimensional state and a linear model is derived for the dynamics. Three different models are developed based on density data from the TIE-GCM, NRLMSISE-00 and JB2008 thermosphere models and are valid from 100 to maximum 800 km altitude. Using the models and TLE data the global density is estimated by simultaneously estimating the density and the orbits and ballistic coefficients of several objects using a Kalman filter. The sequential estimation provides both estimates of the density and corresponding uncertainty. Accurate density estimation using the TLEs of 17 objects is demonstrated and validated against CHAMP and GRACE accelerometer-derived densities. The estimated densities are shown to be significantly more accurate and less biased than NRLMSISE-00 and JB2008 modelled densities. The uncertainty in the density estimates is quantified and shown to be dependent on the geographical location, solar activity and objects used for estimation. In addition, the data assimilation capability of the model is highlighted by assimilating CHAMP accelerometer-derived density data together with TLE data to obtain more accurate global density estimates. Finally, the dynamic thermosphere model is used to forecast the density.

Plain Language Summary

The trajectory of satellites that orbit the Earth at low altitudes (below 1000 km) is affected by drag caused by the Earth’s upper atmosphere. To accurately compute the effect of the drag on the orbit, the mass density of the atmosphere needs to be known. This density can however change quickly due to changing solar activity. To model the changes in the upper atmosphere, we developed a computationally-inexpensive numerical model. Using the model and observations of satellite orbits, we derive information about the atmospheric density. These estimated densities can be used for improving atmospheric and orbit predictions.

1 Introduction

Accurate knowledge of the thermospheric density is essential for orbit prediction in low Earth orbit and in particular for conjunction assessments. The models of the thermosphere with most potential for good forecast capabilities, in particular during storm conditions, are physics-based models (Sutton, 2018), such as the Global Ionosphere-Thermosphere Model (GITM) (Ridley et al., 2006) and the Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM) (Qian et al., 2014). These models solve the continuity, momentum and energy equations for a number of neutral and charged components. Their modeling and prediction performance comes, however, at a high computational cost. The models are very high-dimensional, solving Navier-Stokes equations over a discretized spatial grid involving $10^4$-$10^6$ state variables and 12-20 inputs and internal parameters. In addition, to fully exploit the forecasting potential of physics-based models the schemes employed for data assimilation need to be improved (Sutton, 2018).

Empirical models (Jacchia, 1970; Hedin, 1987; Picone et al., 2002; Bowman et al., 2008; Bruinsma, 2015), on the other hand, capture the average behaviour of the atmosphere using low-order, parameterized mathematical formulations based on historical observations. A major advantage of empirical models is that they are fast to evaluate, making them ideal for drag and orbit computations. The accuracy of these empirical models is however limited (He et al., 2018), especially during space weather events. Improved densities can be obtained by calibrating empirical density models using satellite data.
The current Air Force standard is the High Accuracy Satellite Drag Model (HASDM) (Storz et al., 2005), which is an empirical model that is calibrated using observations of calibration satellites. These satellite observations are used to determine atmospheric model parameters based on their orbit determination solutions. Due to the lack of access to space surveillance observations, publicly available two-line element (TLE) data have been used in the past to estimate the thermospheric density (Picone et al., 2005; Emmert et al., 2006) and calibrate empirical models (Cefola et al., 2004; Yurasov et al., 2005; Doornbos et al., 2008; Chen et al., 2019). Cefola et al. (2004) and Yurasov et al. (2005) calibrated the GOST and NRLMSISE-00 density models using two scaling parameters based on the fitted ballistic coefficient (BC) values. Doornbos et al. (2008) estimated spherical harmonics coefficients to calibrate NRLMSISE-00 model using TLE-derived density estimates and Sang et al. (2011) and Chen et al. (2019) adjusted the 187 coefficients of the DTM87 density model directly during orbit determination of multiple objects. On the other hand, Crowley and Pilinski (2017) used TLE data for data assimilation in the physics-based Dragster model using Ensemble Kalman filtering. Except for the Dragster model, the calibrated empirical models have limited forecasting capability which reduces their effectiveness for orbit prediction.

Recently, a new methodology for modelling and estimating the thermosphere using reduced-order modeling was developed by Mehta and Linares (2017) to overcome the high-dimensionality problem of physics-based models. The technique combines the predictive abilities of physics-based models with the computational speed of empirical models by developing a Reduced-Order Model (ROM) that represents the original high-dimensional system using a smaller number of parameters. The order-reduction is achieved using proper orthogonal decomposition (POD) (Golub & Reinsch, 1970; Rowley et al., 2004), also known as principal component analysis (PCA) (Jolliffe, 2011), empirical orthogonal functions (EOF) or Karhunen-Loeve expansion (Loeve, 1977). The POD approach uses singular value decomposition (SVD) to compute spatial modes that are orthogonal with respect to each other. Using the POD modes, a ROM can be constructed from experimental or numerical data. In addition, POD modes have been used by Matsuo and Forbes (2010) and Sutton et al. (2012) to study and model variations of the thermospheric density. To model the dynamic thermosphere, a dynamic ROM was developed by Mehta et al. (2018) by determining the best fit linear dynamical system from density data using the recently developed Dynamic Mode Decomposition (DMD) technique (Schmid, 2010). The DMD approach assumes a linear system $x_{k+1} = Ax_k$, where $x_k$ is the $k^{th}$ data sampled from a sequential dataset and $A$ is the unknown system matrix. In particular, Dynamic Mode Decomposition with control (DMDc) (Proctor et al., 2016) can include the effect of control and extends the DMD approach to systems with the form $x_{k+1} = Ax_k + Bu_k$ where $u_k$ is the system input. The DMDc method was used by Mehta et al. (2018) to develop a quasi-physical dynamic ROM for the thermosphere. The application of the ROM approach for atmospheric density estimation was demonstrated by data assimilation of accelerometer derived mass density (Mehta & Linares, 2018b) and simulated GPS measurements (Mehta & Linares, 2018a) using Kalman filters. This technique enables both the accurate estimation of thermospheric density and forecasting of the future density through the ROM dynamic model.

The benefits of this approach are that: 1) the modes provide an optimal low-order representation and therefore capture most variance of the system; 2) the global density and uncertainty can be estimated in real-time thanks to the dynamic model, which is not possible with a static model; 3) the dynamic model can be used for forecasting (for static models only the fitted coefficients can be extrapolated in time).

In this work, the reduced-order modelling technique for density estimation is further developed and TLE data is used to estimate the thermospheric density. The availability of TLE data for thousands of objects make them attractive for density estimation; however, the use of TLEs is challenging due to the limited accuracy of the orbital
data. The density estimation using TLE data is achieved by simultaneously estimating
the orbits and BCs of several objects and the reduced-order density state using an un-
scented Kalman filter. The main contributions of the paper are:

1. Nonlinear space weather inputs are introduced to improve ROM prediction.
2. Two new ROM models based on the NRLMSISE-00 and JB2008 models are de-
developed to extend the maximum altitude to 800 km.
3. Modified equinoctial elements are employed to express the orbit and measurements
to retain “Gaussianity” of the state probability density function.
4. Accurate thermospheric density estimates are computed by assimilating TLE data
in reduced-order density models.
5. Accurate density estimation over extended periods of time using a limited amount
of TLE data during both high and low solar activity is demonstrated.
6. The estimated densities are validated against CHAMP and GRACE accelerometer-
derived density data.
7. Improved global density estimates are obtained by assimilating CHAMP accelerometer-
derived density data together with TLE data.

The paper is structured as follows. First the development of a dynamic reduced-
order density model described. After that, the estimation of the density via TLE data
assimilation using an unscented Kalman is discussed. Then the performance of the ROM
density estimation is assessed using simulated and real TLE data, and the uncertainty
quantification and prediction capability of the model are demonstrated. Finally, the re-
sults are discussed and conclusions are drawn.

2 Methodology
The neutral density estimation approach consists of two main components: 1) the
development of a dynamic reduced-order model (ROM) for the thermosphere and 2) the
calibration of the ROM through assimilation of TLE data.

2.1 Reduced-order modeling
The main idea of reduced-order modeling is to reduce the dimensionality of the state
space while retaining maximum information. In our case, the full state space consists
of the neutral mass density values on a dense uniform grid in latitude, local solar time
and altitude. The goal is to develop a model for the density evolution over time. First,
to make the problem tractable, the state space dimension is reduced using POD. Sec-
ond, a linear dynamic model is derived by applying DMDc.

2.1.1 Proper orthogonal decomposition
The concept of order reduction using POD is to project the high-dimensional sys-
tem and its solution onto a set of low-dimensional basis functions or spatial modes, while
capturing the dominant characteristics of the system. Consider the variation \( \tilde{x} \) of the
neutral mass density \( x \) with respect to the mean value \( \bar{x} \):

\[
\tilde{x}(s, t) = x(s, t) - \bar{x}(s)
\]

where \( s \) is the spatial grid. A significant fraction of the variance \( \tilde{x} \) can be captured by
the first \( r \) principal spatial modes:

\[
\tilde{x}(s, t) \approx \sum_{i=1}^{r} c_i(t) \Phi_i(s)
\]

where \( \Phi_i \) are the spatial modes and \( c_i \) are the corresponding time-dependent coefficients.
The spatial modes \( \Phi \) are computed using a SVD of the snapshot matrix \( X \) that contains
\[ X = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \cdots & \tilde{x}_m \end{bmatrix} = U \Sigma V^T \]  

(3)

where \( m \) is the number of snapshots. The spatial modes \( \Phi \) are given by the left singular vectors (the columns of \( U \)). The state reduction is achieved using a similarity transform:

\[ z = U_r^{-1} \tilde{x} = U_r^T \tilde{x} \]  

(4)

where \( U_r \) is a matrix with the first \( r \) POD modes and \( z \) is our reduced-order state. Projecting \( z \) back to the full space gives approximately \( \tilde{x} \) that allows us to compute the density:

\[ x(s, t) \approx U_r(s) z(t) + \bar{x}(s) \]  

(5)

More details on POD can be found in Mehta and Linares (2017).

### 2.1.2 Dynamic Mode Decomposition with control

To enable prediction of the atmospheric density, we develop a linear dynamic model for the reduced-order state \( z \). First, let’s consider the full-dimensional case. Since the atmosphere is highly sensitive to the solar activity, we derive a linear system that considers exogenous inputs:

\[ x_{k+1} = Ax_k + Bu_k \]  

(6)

where \( u_k \) is the system input, which in our case are the space weather inputs. The dynamic matrix \( A \) and input matrix \( B \) can be estimated from output data using the DMDc algorithm. For this, the outputs of the dynamical system (6) or snapshots, \( x_k \), are rearranged into time-shifted data matrices. Let \( X_1 \) and \( X_2 \) be the time-shifted matrix of snapshots such that:

\[ X_1 = \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}, \quad X_2 = \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}, \quad \Upsilon = \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix} \]  

(7)

where \( m \) is the number of snapshots and \( \Upsilon \) contains the corresponding inputs. The data matrices \( X_1 \) and \( X_2 \) are related (\( X_2 \) is the time evolution of \( X_1 \)) through the model in Eq. (6) such that:

\[ X_2 = AX_1 + B \Upsilon \]  

(8)

The goal now is to estimate \( A \) and \( B \). However, because \( X_1 \) and \( X_2 \) can be extremely large, it is more efficient to perform the DMDc in the reduced-order space:

\[ Z_2 = A_r Z_1 + B_r \Upsilon \]  

(9)

where \( Z_1 = U_r^T X_1 \) and \( Z_2 = U_r^T X_2 \) are the reduced-order snapshot matrices, and \( A_r = U_r^T A U_r \) and \( B_r = U_r^T B \) are the reduced-order dynamic and input matrices.

The above equation is modified such that:

\[ Z_2 = \Xi \Psi \]  

(10)

where \( \Xi \) and \( \Psi \) are the augmented operator and data matrices, respectively:

\[ \Xi \triangleq \begin{bmatrix} A_r & B_r \end{bmatrix} \quad \text{and} \quad \Psi \triangleq \begin{bmatrix} Z_1 \\ \Upsilon \end{bmatrix} \]  

(11)

We now estimate the dynamic and input matrices by minimizing \( ||Z_2 - \Xi \Psi|| \). The augmented operator matrix is then solved for by computing the pseudoinverse of \( \Psi \):

\[ \Xi = Z_2 \Psi^+ \]  

(12)
where the + subscript indicates the pseudoinverse. In MATLAB, this can be easily computed using the backslash operator: 
\[ Z^2 \Psi^+ = (\Psi^+)^T (Z_2^T)^T = (\Psi^T Z_2^T)^T \]. Calculating \( A_r \) and \( B_r \) without first computing \( A \) and \( B \) is an improvement with respect to previous work (Mehta et al., 2018) and is also numerically more stable because the number of entries that needs to be determined for matrix \( A_r \) is much smaller than for \( A \).

The matrices \( A_r \) and \( B_r \) are discrete-time matrices for the discrete reduced-order dynamical system:
\[ z_{k+1} = A_r z_k + B_r u_k \] (13)
that corresponds to the fixed timestep \( T \) used for the snapshots. However, for estimation we require continuous information about the density and therefore need a continuous dynamical model:
\[ \dot{z} = A_c z + B_c u \] (14)
where \( A_c \) and \( B_c \) are the continuous-time dynamic and input matrices, respectively. The continuous-time matrices are obtained by converting the discrete-time matrices using the following relation (DeCarlo, 1989):
\[ \begin{bmatrix} A_c & B_c \\ 0 & 0 \end{bmatrix} = \log \left( \begin{bmatrix} A_r & B_r \\ 0 & I \end{bmatrix} \right) / T \] (15)
where \( T \) is the sample time, i.e. the snapshot resolution.

Using a linear system to approximate the nonlinear thermosphere dynamics results in approximation errors. The linear approximation is valid due to the small time step used for the DMDc algorithm. In addition, the same small time step (one hour) is used during estimation such that the state is corrected hourly by the TLE measurements and the error remains small. We refer the reader to the original papers by Mehta and Linares (2017) and Mehta et al. (2018) for an analysis on the accuracy and validity of the linear model. Furthermore, with respect to Mehta et al. (2018), we have improved the prediction performance of the linear model by including nonlinear space weather inputs, see Sections 2.1.3 and 3.1.

### 2.1.3 ROM density model development

**Density training data** In this work, we have developed three different ROM density models using three different atmospheric models to obtain the snapshot matrices, namely the empirical NRLMSISE-00 (Picone et al., 2002) and Jacchia-Bowman 2008 (JB2008) models (Bowman et al., 2008) and the physics-based TIE-GCM model (Qian et al., 2014). We first defined a spatial grid \( s \) in local solar time, geographic latitude and altitude (we use local solar time instead of longitude as azimuthal coordinate, because the diurnal bulge is stationary in solar local time) and computed the density on this grid for every hour over 12 years (one solar cycle), resulting in over 105,000 snapshots. These snapshots were then used to compute a dynamic ROM model, as described in the previous section, using a reduced order of \( r = 10 \). It should be noted that we computed the variation of the density \( x \) by first taking the log base 10 of the density and then subtracting the mean: \( \tilde{x} = \log_{10} x - \log_{10} \bar{x} \), where \( x \) and \( \bar{x} \) are the density and mean density on the spatial grid.

Details on the spatial grid and 12 year periods applied for generating the ROM models can be found in Table 1. Note that the JB2008 ROM model was computed over the years 1999-2010 instead of 1997-2008, because no continuous space weather data was available in the year 1998. The TIE-GCM density data was obtained via simulation using the TIE-GCM model. The TIE-GCM model has a different grid partitioning than used for the ROM model. Therefore, we interpolated the output of the TIE-GCM simulation data to obtain the partitioning used in this work, see Table 1. In addition, the upper boundary of the TIE-GCM model can vary from \( \sim 450 \) to 700 depending on solar activity, be-
cause it uses a log-pressure scale for the vertical coordinate. Therefore, over a full sol-
lar cycle, the maximum altitude for TIE-GCM-based ROM model is limited to 450 km.
For the lower boundary of the model around 97 km, the Global Scale Wave Model was
used to specify migrating diurnal and semidiurnal tidal fields (Hagan et al., 2001) with
eddy diffusivity according to (Qian et al., 2009). More details on the TIE-GCM simul-
ation settings and inputs can be found in Mehta et al. (2018). An improvement with
respect to previous work is the development of ROM models that are valid above 450
km altitude, which is the limiting altitude for TIE-GCM. The new ROM models based
on NRLMSISE-00 and JB2008 extend up to 800 km altitude, see Table 1.

<table>
<thead>
<tr>
<th>Base model</th>
<th>Local solar time [hr]</th>
<th>Latitude [deg]</th>
<th>Altitude [km]</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Domain Resolution</td>
<td>Domain Resolution</td>
<td>Domain Resolution</td>
<td></td>
</tr>
<tr>
<td>TIEGCM</td>
<td>0, 24</td>
<td>$[-87.5, 87.5]$</td>
<td>9.2</td>
<td>[100, 450]</td>
</tr>
<tr>
<td>NRLMSISE-00</td>
<td>0, 24</td>
<td>$[-87.5, 87.5]$</td>
<td>9.2</td>
<td>[100, 800]</td>
</tr>
<tr>
<td>JB2008</td>
<td>0, 24</td>
<td>$[-87.5, 87.5]$</td>
<td>9.2</td>
<td>[100, 800]</td>
</tr>
</tbody>
</table>

**Space weather inputs** The space weather inputs $u_k$ used in the dynamical model
are taken from the inputs required by the original density models, see second column in
Table 2. In addition to these default inputs, we added the next-hour values for key space
weather indices to improve the DMDc prediction, see third column in Table 2. Finally,
a new innovation in this work is the addition of nonlinear space weather terms, such as
the square of an index, e.g. $ap^2$, or the multiplication of two different indices, e.g. $ap$
$F_{10.7}$, see nonlinear inputs in Table 2. The improvement of the DMDc model due to adding
nonlinear terms will be discussed in the results section.

<table>
<thead>
<tr>
<th>Base model</th>
<th>Standard inputs</th>
<th>Future inputs</th>
<th>Nonlinear inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIE-GCM</td>
<td>$doy, hr, F_{10.7}, F_{10.7}, Kp$</td>
<td>$F_{10.7}, Kp$</td>
<td>$Kp^2, Kp \cdot F_{10.7}$</td>
</tr>
<tr>
<td>NRLMSISE-00</td>
<td>$doy, hr, F_{10.7}, F_{10.7}, ap^\dagger$</td>
<td>$F_{10.7}, F_{10.7}, ap^\dagger$</td>
<td>$ap^2, ap_{now} \cdot F_{10.7}$</td>
</tr>
<tr>
<td>JB2008</td>
<td>$doy, hr, F_{10.7}, F_{10.7}, S_{10}, S_{10}, M_{10}, M_{10}, Y_{10}, DSTDTC, GMST, a_{SUN}, \delta_{SUN}$</td>
<td>$F_{10.7}, S_{10}, M_{10}, Y_{10}, DSTDTC$</td>
<td>$DSTDTC^2, DSTDTC \cdot F_{10.7}$</td>
</tr>
</tbody>
</table>

† $ap$ indices for the NRLMSISE-00 model consist of 8 $ap$ values for up to 57 hours prior to current time

2.2 Density estimation

The neutral mass density is estimated through the assimilation of two-line element
orbital data in the dynamic ROM model. This is achieved by simultaneously estimating
the ROM state and the orbit and BC of objects using an unscented Kalman filter.

2.2.1 Two-line element data

The US Air Force Space Command publicly distributes the orbital data of thou-
sands of Earth-orbiting objects in the form of two-line element sets. From this TLE data,
the state of an object (position and velocity) at any epoch can be extracted using the SGP4/SDP4 models (Hoots & Roehrich, 1980; Vallado et al., 2006). Hence, the effect of drag can be observed in TLE orbital data if the drag perturbation is strong enough.

A general concern when using TLE data is the accuracy of the orbital data. In the SGP4/SDP4 models only the largest perturbations are included, while higher-order and short-periodic effects, a dynamic atmosphere and orbital maneuvers are not accounted for (Vallado & Cefola, 2012). As a result, there can be large errors in the orbital data (Kelso, 2007). In addition, since 2013, TLEs are fitted to a higher-order orbital solution that includes a future orbit prediction (Hejduk et al., 2013). This generally improve the TLE accuracy at epoch, but may deteriorate the quality if inaccurate future space weather is used for the orbit prediction. To gain understanding about errors in TLE data, we compared the position according to TLE data against GPS data, see Figure 1. For this, we used the GPS data of a Planet Labs satellite at 494 km altitude (more information on the Planet Labs ephemeris can be found in Foster et al. (2015) and http://ephemerides .planet-labs.com). The position error is largest in the along-track direction and varies with a 12-hour period, which is thought to be due to missing tesseral m-daily terms in the SGP4 theory (Herriges, 1988). From the figure, it is clear that we can expect significant errors in the orbital data. On the other hand, the errors are expected to be limited when computing the state at epochs prior to the TLE epoch (see top plot in Figure 1), which corresponds with the period of tracking data used to generate the TLE.

The objects used for density estimation need to be selected carefully. First of all, the objects preferably have a strong drag signal. In particular, the effect of drag should be strong with respect to other non-conservative force effects, else errors in non-conservative force modelling, such as solar radiation pressure, can result in inaccurate density estimates. Second, it is important to have an accurate estimate of the true BC of the object to minimize errors due to inaccuracies in the BC. Therefore, the variation of the BC over time should preferably be very small or else must be modelled accurately (Bowman, 2002). An overview of the objects used in this work is shown in Table 3. The BC values were taken from Bowman et al. (2004), Emmert et al. (2006) and Lu and Hu (2017). A 1-σ uncertainty of 1% was assumed for the BC values. This is smaller than the 5-10% error often assumed for the drag coefficient (Emmert et al., 2006), but close to the 2-3% accuracy found for 30-year averaged BC estimates (Bowman et al., 2004). Finally, any orbit maneuvers and significant outliers in the TLE data must be detected and excluded.

Figure 1. TLE position error with respect to GPS data for a satellite at 494 km altitude using a single TLE (top) or multiple TLEs (bottom) in 8-day window in Jan 2018.
Table 3. Objects used for density estimation from 2002 to 2008. The BC values were taken from Bowman et al. (2004), Emmert et al. (2006) and Lu and Hu (2017).

<table>
<thead>
<tr>
<th>NORAD Catalog ID</th>
<th>Object</th>
<th>BC [m²/kg]</th>
<th>Perigee height [km]</th>
<th>Apogee height [km]</th>
<th>Inclination [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>Tiros 2</td>
<td>0.01486</td>
<td>509 - 460</td>
<td>555 - 493</td>
<td>48.5</td>
</tr>
<tr>
<td>165</td>
<td>DELTA 1 R/B</td>
<td>0.05326</td>
<td>598 - 532</td>
<td>620 - 550</td>
<td>47.9</td>
</tr>
<tr>
<td>614</td>
<td>Hitchhiker 1</td>
<td>0.01453</td>
<td>319 - 312</td>
<td>2061 - 1707</td>
<td>82.0</td>
</tr>
<tr>
<td>2153</td>
<td>THOR AGENA B</td>
<td>0.03329</td>
<td>501 - 497</td>
<td>2636 - 2598</td>
<td>79.7</td>
</tr>
<tr>
<td>2622</td>
<td>OV1-9 R/B</td>
<td>0.02240</td>
<td>475 - 471</td>
<td>4500 - 4468</td>
<td>99.1</td>
</tr>
<tr>
<td>4221</td>
<td>Azur</td>
<td>0.02201</td>
<td>368 - 362</td>
<td>1817 - 1707</td>
<td>82.0</td>
</tr>
<tr>
<td>6073</td>
<td>Cosmos 482 Debris</td>
<td>0.00378</td>
<td>213 - 206</td>
<td>4985 - 3984</td>
<td>52.1</td>
</tr>
<tr>
<td>7337</td>
<td>Vektor</td>
<td>0.01120</td>
<td>380 - 372</td>
<td>1429 - 1328</td>
<td>82.9</td>
</tr>
<tr>
<td>8744</td>
<td>Vektor</td>
<td>0.01117</td>
<td>380 - 372</td>
<td>1429 - 1328</td>
<td>82.9</td>
</tr>
<tr>
<td>12138</td>
<td>Vektor</td>
<td>0.01115</td>
<td>394 - 378</td>
<td>1596 - 1519</td>
<td>83.0</td>
</tr>
<tr>
<td>12388</td>
<td>Vektor</td>
<td>0.01121</td>
<td>384 - 379</td>
<td>1555 - 1460</td>
<td>83.0</td>
</tr>
<tr>
<td>14483</td>
<td>Vektor</td>
<td>0.01130</td>
<td>390 - 385</td>
<td>1658 - 1581</td>
<td>82.9</td>
</tr>
<tr>
<td>20774</td>
<td>Vektor</td>
<td>0.01168</td>
<td>391 - 387</td>
<td>1764 - 1684</td>
<td>83.0</td>
</tr>
<tr>
<td>23278</td>
<td>Vektor</td>
<td>0.01168</td>
<td>398 - 394</td>
<td>1851 - 1783</td>
<td>83.0</td>
</tr>
<tr>
<td>26405</td>
<td>CHAMP</td>
<td>0.00477†</td>
<td>400 - 314</td>
<td>434 - 318</td>
<td>87.2</td>
</tr>
<tr>
<td>27391</td>
<td>GRACE 1</td>
<td>0.00697</td>
<td>480 - 447</td>
<td>505 - 470</td>
<td>89.0</td>
</tr>
<tr>
<td>27392</td>
<td>GRACE 2</td>
<td>0.00693</td>
<td>480 - 447</td>
<td>506 - 470</td>
<td>89.0</td>
</tr>
</tbody>
</table>

† Average of values reported by Emmert et al. (2006) and Lu and Hu (2017)

from the data before estimation. The TLE time series used in this work do not contain any orbit maneuvers and contain only few outliers. These outliers are small and were found to have little effect on the density estimation results; therefore, the unfiltered TLE data was used for estimation (the largest outlier was found in the TLE data of object 2153 causing a several-km deviation in the orbit for a couple of days; removing the outliers can result in larger orbit errors due to a gap in the TLE time series).

2.2.2 Unscented Kalman filter

To fuse the model and TLE data, we use the square-root unscented Kalman filter (UKF). The UKF uses an unscented transform (UT) to avoid large errors in the true posterior mean and covariance of the variables. Here, the true posterior mean and covariance are computed to the 3rd order by propagating a carefully selected set of sample points, called sigma points, through the true nonlinear dynamics. The UKF is a popular algorithm that is well documented in literature; therefore, for details about the square-root UKF the reader is referred to Wan and Van Der Merwe (2001). The state, dynamics, measurements and noise used to estimate the density with the UKF are described in the following.

2.2.2.1 State The state \( \mathbf{x} \) that is estimated in the UKF consists of the osculating orbital states (expressed in modified equinoctial elements) and the BCs of the objects plus the reduced-order density state \( \mathbf{z} \):

\[
\mathbf{x} = \begin{bmatrix} p_1, f_1, g_1, h_1, k_1, L_1, BC_1, & \ldots, & p_n, f_n, g_n, h_n, k_n, L_n, BC_n, & \mathbf{z}^\top \end{bmatrix}^\top \tag{16}
\]

where \( n \) is the number of objects and the modified equinoctial elements (MEE) are defined as (Walker et al., 1985):

\[
p = a(1 - e^2), \quad f = e \cos(\omega + \Omega), \quad g = e \sin(\omega + \Omega),
\]

\[
h = \tan(i/2) \cos\Omega, \quad k = \tan(i/2) \sin\Omega, \quad L = \Omega + \omega + \nu. \tag{17}
\]

where \( a, e, i, \Omega, \omega \) and \( \nu \) are the classical Keplerian orbital elements. The MEE are non-singular and tend to behave less nonlinear than the Cartesian coordinates (used in previous work), which benefits the Kalman filter estimation by mitigating departure from

---
“Gaussianity” of the propagated state probability density function (PDF) (Poore, 2015, Section 7.2). To initialize the state, we use the objects’ orbital states according to TLE data and take the BC values from Table 3. The ROM state is initialized using densities from the JB2008 model.

2.2.2.2 Dynamic model The dynamic model $f(x,t)$ for evolving the state $x$ consists of propagating the orbital states using orbital dynamics and evolving the ROM state using the continuous-time DMDc model:

$$\dot{x} = f(x,t) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \\ BC \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ a_{grav,x} + a_{drag,x} \\ a_{grav,y} + a_{drag,y} \\ a_{grav,z} + a_{drag,z} \\ 0 \\ A_z + B_z \end{bmatrix}$$

where $[x,y,z]$ and $[v_x,v_y,v_z]$ are the inertial position and velocity, respectively, that are used for orbit propagation and $BC = \frac{C_d A}{m}$ is the ballistic coefficient. The ROM state $z$ is used to compute the atmospheric density by converting $z$ to the full space (see Eq. 5) and interpolating the density grid. The orbital dynamics are expressed in inertial Cartesian coordinates whereas the orbital states estimated in the UKF are expressed in MEE to retain “Gaussianity” of the state PDF. For sigma point propagation in the UKF, the sigma points and weights are first selected in MEE space. Each sigma point is then transformed to Cartesian space and propagated to the next epoch. Finally, each sigma point is transformed back to MEE space and the state and covariance are computed based on the MEE sigma points and weights.

2.2.2.3 Orbital dynamics The orbital dynamics used in this work considers:

- Geopotential acceleration computed using the EGM2008 model, up to degree and order 48 for the harmonics;
- Solar radiation pressure with dual-cone shadow model;
- Third body perturbations from Sun and Moon;
- Atmospheric drag considering a rotating atmosphere for computing the velocity relative to the atmosphere. The atmospheric density is computed using the ROM density model.

According to Bowman et al. (2004), a 48x48 geopotential field provides acceptable accuracy for density estimation. For solar radiation pressure, we assumed a reflectivity coefficient of 1.2 and extracted the area-to-mass ratio from the object’s BC assuming a drag coefficient of 2.2. The orbit propagation is carried out in the inertial J2000 reference frame using Cartesian position and velocity while the geopotential and drag accelerations are computed in the Earth-fixed ITRF93 frame. NASA’s SPICE toolbox is used both for Moon and Sun ephemerides (DE405 kernels) and for reference frame and time transformations (ITRF93 and J2000 reference frames and leap-seconds kernel).

2.2.2.4 Measurements The measurements used for estimation are the osculating orbital states extracted from TLE data. At one hour intervals the osculating state of each object is computed using the nearest newer TLE by propagating the TLE backward to the measurement epoch using SGP4. These states are then converted to MEE and used as measurements. The 17 objects used in this work for density estimation are shown in Table 3. Note that for calibrating the ROM-TIEGCM model only 11 objects are used, because 6 of the 17 objects have their perigee above the ROM-TIEGCM maximum altitude of 450 km.
2.2.2.5 Measurement and process noise  The measurements noise $R$ was determined empirically by comparing the post-fit orbits with the measurements. The measurement noise covariance $R$ was chosen such that the post-fit residuals are tightly within the 3-σ range. The post-fit residuals in $p$, $f$ and $g$ were found to increase with increasing eccentricity; therefore, the variance for the measurements of $p$, $f$ and $g$ was scaled by the orbit’s eccentricity $e$ to obtain the following eccentricity-dependent measurement noise:

$$
\begin{bmatrix}
    R_p, R_f, R_g, R_h, R_k, R_L
\end{bmatrix} = 
\begin{bmatrix}
    c_1 \cdot 10^{-8}, c_2 \cdot 10^{-10}, c_2 \cdot 10^{-10}, 10^{-9}, 10^{-9}, 10^{-8}
\end{bmatrix}
$$

(19)

where $c_1 = 1.5 \cdot \max(4e, 0.0023)$ and $c_2 = 3 \cdot \max(e/0.004, 1)$.

The process noise variance $Q$ for the state and BC was set to:

$$
\begin{bmatrix}
    Q_p, Q_f, Q_g, Q_h, Q_k, Q_L, Q_{BC}
\end{bmatrix} = 
\begin{bmatrix}
    1.5 \cdot 10^{-8}, 2 \cdot 10^{-14}, 2 \cdot 10^{-14}, 10^{-14}, 10^{-14}, 10^{-16}
\end{bmatrix}
$$

(20)

The process noise for the ROM state $Q_z$ was computed using the 1-hour ROM prediction error on the training data:

$$
\text{diag}(Q_z) = \text{diag} (\text{Cov}[Z_2 - (A, Z_1 + B, Y)])
$$

(21)

As a result of this approach, the Kalman filter will give more confidence to the model prediction with respect to measurements when the ROM prediction on the training data is more accurate. These values for the measurements and process noise were found to give good estimated density results (i.e. low RMS error of the estimated density) and good estimates of the uncertainty in the estimated density. If the measurement noise is set too low and the process noise too high, the density estimates will soak up errors in the TLE data. On the other hand, if the measurement noise is set too high and the process noise too low, the filter will give too much confidence to the model prediction with respect to measurements. Finally, the initial covariance for the state was set to:

$$
\begin{bmatrix}
    P_p, P_f, P_g, P_h, P_k, P_L, P_{BC}, P_{z_1}, P_{z_n}
\end{bmatrix} = 
\begin{bmatrix}
    R_p, R_f, R_g, R_h, R_k, R_L, (0.01 \cdot BC)^2, 20, 5
\end{bmatrix}
$$

(22)

where $P_{z_1}$ refers to the covariance for the first reduced-order mode $z_1$ and $P_{z_n}$ to the covariance for all other modes. An overview of the reduced-order model density estimation technique is shown below.

---

**Algorithm 1 ROM density estimation**

**Reduced-order modeling**
1. Generate density training data $X$ (hourly density on grid) using physics-based or empirical density model
2. Select reduced order $r$
3. Compute reduced-order model using a SVD of the snapshots $X$ (Eqs. 1-4)
4. Compute DMDc for reduced-order training data (Eqs. 7-15)

**Density estimation**
5. Download TLE data and estimate true BC
6. Select objects with accurate TLEs (check self-consistency) and stable BC (not maneuvering)
7. Generate measurements (orbital states in MEE) every hour from TLEs using SGP4 using nearest newer TLE
8. Estimate ROM modes $z$ using unscented Kalman filter by simultaneously estimating the modes and the state and BC of objects

---

3 Results

In this section, the performance of the ROM model forecasting and density estimation using TLE data is assessed.
3.1 ROM density prediction

The performance of the dynamic ROM models is tested by comparing density forecasts with training data. Using the three different ROM density models the density was predicted for 5 days during quiet space weather conditions and during a geomagnetic storm in 2002. The resulting density forecast errors (the root mean square (RMS) percentage error on the three-dimensional spatial grid) and space weather conditions are shown in Figure 2. The predictions using the ROM model based on JB2008 are most accurate. This good performance can be explained by the superior space weather proxies used by the ROM-JB2008 model. Table 4 shows the average 1-hour, 1-day and 3-days prediction accuracy of the ROM models with respect to the training data. The table shows that the addition of nonlinear space weather terms improves the prediction accuracy for all models. The ROM-JB2008 provides the best predictions with respect to training data with an error of only 3.6% after 3 days. The nonlinear terms especially improve the prediction during a geomagnetic storm, see Figure 2. Finally, the average truncation error due to reducing the dimension to $r = 10$ is between 0.5 and 4%, see Table 4. The small truncation error for the JB2008-based ROM model can be explained by the fact that the JB2008 model uses a simple density distribution (see also Figure 7).

3.2 Simulated TLE test case

To assess whether accurate density estimation using a ROM model and TLE data is feasible, we first tested the technique using simulated TLE data. Mehta and Linares (2018a) already demonstrated the feasibility of accurate density estimation using simulated GPS measurements with a 5-minute resolution. For the TLE scenario, the ‘true’ orbits and densities are first computed by numerically propagating eight objects using...
Table 4. Average prediction error in density using linear or nonlinear space weather inputs and average truncation error due to order-reduction with r = 10 with respect to training data. The errors are computed as the root mean square (RMS) error in density on the three-dimensional spatial grid. The average error over all 105,000 epochs of the training data are shown. The prediction errors include truncation errors due to order-reduction.

<table>
<thead>
<tr>
<th>ROM model</th>
<th>Space weather inputs</th>
<th>Mean prediction error [%]</th>
<th>Mean truncation error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 hour</td>
<td>1 day</td>
</tr>
<tr>
<td>NRLMSISE</td>
<td>Linear</td>
<td>3.81</td>
<td>8.26</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>3.77</td>
<td>6.80</td>
</tr>
<tr>
<td>TIEGCM</td>
<td>Linear</td>
<td>2.23</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>2.21</td>
<td>4.30</td>
</tr>
<tr>
<td>JB2008</td>
<td>Linear</td>
<td>1.32</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>0.90</td>
<td>2.61</td>
</tr>
</tbody>
</table>

Table 5. Initial orbital parameters for the eight simulated true orbits.

<table>
<thead>
<tr>
<th>Object</th>
<th>a [km]</th>
<th>e [-]</th>
<th>i [deg]</th>
<th>Ω [deg]</th>
<th>ω [deg]</th>
<th>M [deg]</th>
<th>BC [m²/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6811.031</td>
<td>3.011E-3</td>
<td>81.208</td>
<td>157.262</td>
<td>106.464</td>
<td>52.070</td>
<td>0.0142</td>
</tr>
<tr>
<td>2</td>
<td>6777.764</td>
<td>1.300E-3</td>
<td>81.225</td>
<td>184.489</td>
<td>329.642</td>
<td>122.045</td>
<td>0.0170</td>
</tr>
<tr>
<td>3</td>
<td>6810.172</td>
<td>1.293E-3</td>
<td>81.215</td>
<td>187.594</td>
<td>112.894</td>
<td>78.318</td>
<td>0.0168</td>
</tr>
<tr>
<td>4</td>
<td>6808.532</td>
<td>5.124E-4</td>
<td>53.014</td>
<td>185.496</td>
<td>112.894</td>
<td>78.318</td>
<td>0.0170</td>
</tr>
<tr>
<td>5</td>
<td>6794.771</td>
<td>2.901E-3</td>
<td>82.094</td>
<td>157.262</td>
<td>106.464</td>
<td>52.070</td>
<td>0.0142</td>
</tr>
<tr>
<td>6</td>
<td>6785.760</td>
<td>4.594E-4</td>
<td>97.435</td>
<td>67.678</td>
<td>86.303</td>
<td>88.988</td>
<td>0.0220</td>
</tr>
<tr>
<td>7</td>
<td>6729.365</td>
<td>1.619E-3</td>
<td>87.251</td>
<td>169.664</td>
<td>52.108</td>
<td>83.135</td>
<td>0.0052</td>
</tr>
<tr>
<td>8</td>
<td>6828.232</td>
<td>1.135E-3</td>
<td>30.411</td>
<td>270.733</td>
<td>29.570</td>
<td>295.859</td>
<td>0.0536</td>
</tr>
</tbody>
</table>

the full dynamics described in Section 2.2.2.3 and the ROM-TIEGCM density model. The initial orbital parameters for the true orbits are shown in Table 5. Based on these ‘true’ orbits, TLE measurement data is simulated assuming Gaussian noise and a 1-hour resolution. The 1-σ errors used for simulated TLE measurements are:

\[
[\sigma_p, \sigma_f, \sigma_g, \sigma_h, \sigma_k, \sigma_L] = [0.045, 2.0 \cdot 10^{-5}, 2.0 \cdot 10^{-5}, 2.0 \cdot 10^{-5}, 2.0 \cdot 10^{-5}, 1.25 \cdot 10^{-4}] \quad (23)
\]

These errors were established empirically based on TLE data of near-circular orbits around 400 km altitude in the years 2017 and 2018. These errors convert to RMS position and velocity errors of 0.88 km and 1.0 m/s, respectively. The initial guesses for estimating the orbits, BCs and ROM-state also include these errors.

Figure 3 shows the errors in the estimated BCs and densities along the orbits during the estimation period. The errors in density and BC remain below 2% after 12 days estimation. If the assumed TLE 1-σ errors are twice as large, then the results are similar but the errors in density and BC after 12 days increase by 0.5–1%, which indicates that accurate density estimation is feasible even when measurement errors are large. In Figure 4 the true and estimated values and errors of the first four ROM modes are shown. All modes converge to their true values, while a small bias in the first mode remains. The convergence of the modes is also correctly displayed by the 3-σ error bounds. These results demonstrate that accurate density estimation using the TLE data of multiple objects is feasible and that both the BC and density are observable. Nevertheless, in reality, less accurate density estimates can be expected, because new TLE measurements are not available every hour and errors in TLE data are not random nor Gaussian.
Figure 3. Error in estimated density (left) and BC (right) for each object in simulated TLE test case since 00:00 UTC on day 191 of year 2005.

Figure 4. True and estimated ROM modes and corresponding error and 3σ error bounds for simulated TLE test case since 00:00 UTC on day 191 of year 2005.
Table 6. Accuracy of estimated and modelled densities along CHAMP’s and GRACE-A’s orbit during August 2002. The numbers show the mean $\mu$, standard deviation $\sigma$ and root-mean-square (RMS) of the error in orbit and daily-averaged density as percentage of true densities.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>Density error [%]</th>
<th>Orbit-averaged</th>
<th>Daily-averaged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>RMS</td>
</tr>
<tr>
<td>CHAMP</td>
<td>NRLMSISE-00</td>
<td>24.2</td>
<td>11.4</td>
<td>26.7</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>-22.4</td>
<td>7.7</td>
<td>23.7</td>
</tr>
<tr>
<td></td>
<td>ROM-TIEGCM</td>
<td>4.6</td>
<td>10.1</td>
<td>11.1</td>
</tr>
<tr>
<td></td>
<td>ROM-NRLMSISE</td>
<td>-2.6</td>
<td>7.3</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>ROM-JB2008</td>
<td>-2.0</td>
<td>6.9</td>
<td>7.2</td>
</tr>
<tr>
<td>GRACE-A</td>
<td>NRLMSISE-00</td>
<td>39.8</td>
<td>16.5</td>
<td>43.1</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>-20.8</td>
<td>10.8</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td>ROM-TIEGCM</td>
<td>12.1</td>
<td>15.3</td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td>ROM-NRLMSISE</td>
<td>4.7</td>
<td>10.0</td>
<td>11.1</td>
</tr>
<tr>
<td></td>
<td>ROM-JB2008</td>
<td>8.4</td>
<td>9.3</td>
<td>12.5</td>
</tr>
</tbody>
</table>

3.3 Real TLE

In the following, the density is estimated using real TLE data and compared with
CHAMP and GRACE accelerometer-derived density data. Figure 5 shows the orbit-averaged
estimated density along CHAMP’s orbit as well as the density according to CHAMP data
and the NRLMSISE-00 and JB2008 density models during August 2002 (the first month
for which we had both CHAMP and GRACE data). All three ROM models perform very
well. Especially, the ROM-NRLMSISE and ROM-JB2008 models are very well able to
estimate density variations due to changes in solar activity. The RMS errors in the daily-
averaged CHAMP density are less than 6% for the ROM-NRLMSISE and ROM-JB2008
models and less than 9% for the ROM-TIEGCM model, see Table 6. The wiggles in the
estimated orbit-averaged density (particularly visible for ROM-TIEGCM and ROM-NRLMSISE)
have a period of 12 hours, which suggests that these are due to errors in the TLEs due
to missing m-dailies. Similar 12-hour variations can be found in the estimated BCs (not
shown here). Further tuning of the measurement and process noise can reduce the am-
plitude of these variations due to TLE errors.

Overall, the ROM model based on JB2008 performs best with a standard devia-
tion in orbit-averaged density of only 6.9% and 9.3% along CHAMP’s and GRACE-A’s
orbit, respectively. This shows the high accuracy and temporal resolution that can be
achieved by the ROM approach using only TLE data. The error in GRACE-A density
is possibly higher because less objects around GRACE’s 480 km altitude were used than
around CHAMP’s altitude of 400 km and because the drag signal is weaker at higher
altitudes. Only 17 objects were used for calibrating the ROM-JB2008 and ROM-NRLMSISE
models and only 11 for the ROM-TIEGCM model. This is significantly lower than the
36 and 48 objects used by Doornbos et al. (2008) and Shi et al. (2015), respectively, but
comparable to the 16 objects used by Yurasov et al. (2005). It should also be noted that
the errors presented in this paper are with respect to accelerometer-derived density data
and not with respect to TLE-derived density data as in some other works (Doornbos et
al., 2008; Shi et al., 2015).

A close-up of the density along CHAMP’s orbit on August 14, 2002 is shown in Fig-
ure 6. In this time window, the ROM-estimated densities are very close to the true den-
sity (both the mean and variation are estimated well). It is probably not feasible to ex-
Figure 5. Orbit-averaged density along CHAMP orbit according to ROM estimation, CHAMP data, and JB2008 and NRLMSISE-00 models from August 1 to 31, 2002.

Figure 6. Density along CHAMP orbit according to ROM estimation, CHAMP data, and JB2008 and NRLMSISE-00 models (day 226 of year 2002 is August 14, 2002).
Figure 7. Maps of modeled density at 450 km latitude on August 8, 2002 at 0:00:00 UTC.

3.3.1 Uncertainty

Figures 8 and 9 show the uncertainty in the estimated density for different altitudes, latitudes and local solar time. The uncertainty in the estimated density is smaller for lower altitude and inside the diurnal bulge. This indicates that the density estimation is more accurate when the drag signal is stronger. In addition, Figure 9 shows that the uncertainty grows little at altitudes where measurements are available. The estimated 1-σ uncertainties at 400 and 500 km altitude of about 6.5% and 7.5%, respectively, are close to the actual standard deviation of the estimated density at 416 km (CHAMP) and 502 km (GRACE-A) altitude of 6.9% and 9.3%, respectively, see Table 6. Furthermore, between 100 and 800 km altitude the 1-σ error varies between 3 and 11%, while Mehta and Linares (2018b) found a 1-σ error of 5% along CHAMP’s orbit and up to 25% error at higher altitudes after assimilating CHAMP accelerometer-derived density data in a TIEGCM-based ROM model. This indicates that the use of data from multiple objects improves the global density estimates.
Figure 8. 1-σ uncertainty in density estimated using ROM-JB2008 at 300 to 600 km altitude after 30 days estimation on August 31, 2002.

Figure 9. 1-σ uncertainty in density estimated using ROM-JB2008 at two locations for varying altitude after 30 days estimation on August 31, 2002.
Figure 10. Error in orbit-averaged density with respect to CHAMP data for ROM-JB2008 estimated density, and JB2008 and NRLMSISE-00 models for the year 2007.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>Density error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2003</td>
<td>2007</td>
</tr>
<tr>
<td></td>
<td>$\mu$  $\sigma$</td>
<td>$\mu$  $\sigma$</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>RMS</td>
</tr>
<tr>
<td>CHAMP</td>
<td>NRLMSISE-00 4.4 19.7 20.2 26.4 15.4 30.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JB2008 -1.9 12.5 12.7 9.5 13.4 16.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROM-JB2008 -5.1 11.4 12.5 -6.9 9.0 11.4</td>
<td></td>
</tr>
<tr>
<td>GRACE-A</td>
<td>NRLMSISE-00 20.7 28.6 35.3 44.1 28.4 52.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JB2008 10.0 18.3 20.9 16.0 28.4 32.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROM-JB2008 6.5 16.9 18.1 -2.5 21.9 22.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Accuracy of estimated and modelled densities along CHAMP’s and GRACE-A’s orbit over the year 2003 and 2007. The numbers show the mean $\mu$, standard deviation $\sigma$ and root mean square (RMS) of the error in orbit-averaged density as percentage of true densities.

3.3.2 Full years 2003 and 2007

The neutral mass density was estimated using the ROM-JB2008 model over the entire years 2003 (high solar activity) and 2007 (low solar activity). The difference in the estimated and true densities along CHAMP’s and GRACE-A’s orbits are shown in Table 7 and Figure 10. Both the ROM estimation and JB2008 model perform very well in 2003, whereas the ROM estimates are much more accurate than the JB2008 and NRLMSISE-00 models in 2007. Table 7 shows that the ROM-estimated densities are less biased and have a smaller variance than the empirical models. In particular in 2007, when the empirical models are least accurate, the ROM densities have a significantly smaller bias and standard deviation. Furthermore, the estimated density along GRACE-A’s orbit are less accurate. This agrees with the increasing uncertainty in estimated density (see Figure 9) and weaker drag signal at higher altitudes. The accuracy can be further improved by using more accurate orbital data and by improving the spatial and temporal coverage of the measurements, see Section 3.4.

3.3.3 Geomagnetic storm

Figure 11 shows the density along CHAMP’s orbit estimated using the ROM-JB2008 model during a major geomagnetic storm in 2003. Both the ROM and empirical models provide good density estimates during the storm. However, after storm on day 151 the ROM estimates are much more accurate than the empirical models, which overestimate the density, due to calibration. This example shows that the linear ROM model is able to deal with space weather events even though these events are strongly nonlinear.
Figure 11. Orbit-averaged density along CHAMP’s orbit according to ROM estimation, CHAMP data, and JB2008 and NRLMSISE-00 models during major geomagnetic storm on May 30, 2003 (Kp up to 8.3).

Figure 12. Orbit-averaged density along GRACE-A’s orbit estimated using only TLE data or using TLE data and CHAMP accelerometer-derived density data (Feb 16 to Mar 10, 2007).

3.4 Including density data

For some periods of time, one may have access to highly-accurate density data, such as CHAMP, GRACE or GOCE accelerometer-derived densities. This data can be included in the data assimilation to improve the global density estimates. Figure 3.4 shows the estimated orbit-averaged density along GRACE-A’s orbit after assimilating CHAMP density data together with TLE data (here CHAMP was at 360 km and GRACE-A at 480 km altitude; we assumed a 1-σ error in the CHAMP density data of 5%). This period coincides with the period of reduced estimation accuracy shown in Figure 10 around day 50. One density measurement is included every hour at the same time as the TLE orbit measurements. The inclusion of the CHAMP densities significantly improves the GRACE density estimates; the error in daily-averaged density reduced from 16.4% to 11.6%. Therefore, global density estimates can be improved by including accurate density data. This is especially useful in case the drag signal is weak and consequently the density is difficult to observe from orbital data such as during periods of low solar activity.
3.5 Density forecast

The density along CHAMP’s orbit was predicted for 11 days using the ROM-NRLMSISE model, see Figure 13. The initial ROM state used for the prediction was obtained after 30 days calibration in August 2002, see Figure 5. The ROM does very well in predicting the density, even during two geomagnetic storms. This example shows that ROM models are able to accurately forecast the future density if the initial state of the thermosphere and the future space weather are accurately known. In future work, prediction of the future space weather will also be considered.

4 Discussion

The key motivation for the use of TLE data is the widespread availability of this data source. However, TLE orbital data can have significant errors. The presented results indicate that these errors do not limit the ability to use TLE data for density calibration. In addition, Figure 3.4 showed that the use of more accurate measurements can further improve the estimation of the global density. Therefore, in the future other data sources, such as radar and GPS orbital data, could be used to obtain improved density estimates.

To improve the predictive performance of the linear ROM models and therefore of the density estimates, nonlinear space weather inputs were used. This enhancement improved the modelling of the relation between space weather and thermospheric density, which is important because the thermosphere is strongly forced by solar activity. In particular during space weather events, such as geomagnetic storms, the predictive capability of the ROM model is important because the temporal resolution of TLE data is limited. In addition, in future work, the reduced-order models can be further improved by using more accurate density training data for deriving the models using POD and DMDc.

In addition to estimating the global density, we obtained estimates of the uncertainty in the density, see Figures 8 and 9. This information is very valuable for uncertainty quantification, which is needed for, e.g., conjunction assessments. These uncertainty estimates are in particular important for short-term predictions when errors in the density prediction are dominated by errors in the now-cast, whereas after several days errors due to inaccurate space weather forecasting become dominant.
Finally, the ballistic coefficients in Table 3 may have biases that originate from the atmospheric models used in the BC estimation. Such biases may result in inaccuracies in the density estimates. The presented methodology counteracts this by simultaneously estimating the thermospheric density and ballistic coefficients. Still, some error in BC may be soaked up by the density estimates. These errors can be reduced by using the geometry models of the satellites and by modeling the gas-surface interactions and physical drag coefficient to compute the BC (Mehta et al., 2017) instead of estimating the BC from orbital data. This approach can be applied in future work to further debias the density and BC estimates.

5 Conclusions

In this paper we presented a new approach for thermospheric density estimation using TLE data. To obtain accurate global density estimates we assimilated the TLE data of spatially-spread objects in a dynamic reduced-order density model by simultaneously estimating the orbits, BCs and density. The estimated densities were compared with NRLMSISE-00 and JB2008 modelled densities and CHAMP and GRACE-A accelerometer-derived densities. Even though TLE orbital data have significant errors and low temporal resolution, the density estimates are accurate and have a significantly smaller bias and smaller standard deviation than densities computed using the empirical models both during periods of high and low solar activity.

To accurately model the density, we developed three different ROM models based on TIE-GCM, NRLMSISE-00 and JB2008 density data with an upper altitude up to 800 km and improved the prediction performance of the models by including nonlinear space weather terms as inputs for the ROM dynamics. In addition, the estimation using an UKF was improved by expressing the measurements in modified equinoctial elements and by calculating the process noise for the ROM model based on training data performance. We obtained improved global density estimates by including accurate density measurements in addition to TLE data. This indicates that more accurate observations, such as radar and GPS orbital data, could be used in the future to obtain improved density estimates.

Furthermore, the ROM model was shown to be able to provide accurate density forecasts. Therefore, the density estimates and predictions can be used for both improved orbit determination and orbit prediction. Moreover, the technique provides uncertainty estimates for the density, which can be used for uncertainty quantification for, e.g., conjunction assessment.

Future work will focus on further improving the ROM dynamic models to deal with nonlinearities by improving the choice of space weather inputs and by applying Koopman operator theory. In addition, we can estimate the global thermospheric neutral density using historic TLE data from the 1960’s up to present time to generate a density database.

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derived by Sutton (2008) and can be found, e.g., on http://tinyurl.com/densitysets. The MATLAB code and ROM models used in this work for density estimation with TLE data will be published after acceptance on https://zenodo.org including a Digital Object Identifier (DOI).

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